Term Paper PHY 235w

Snowboarding the Halfpipe with Classical Mechanics

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Abstract: The goal of this report is to characterize the path and motion that a snowboarder should carry out to achieve the best score and performance on the halfpipe. Performance on the halfpipe is primarily a function of total time spent in the air and total degree of rotation. I look at what path the rigid body snowboarder should take in order to maximize his air time, as well as what the snowboarder must do in order to perform tricks in the air. I find that the snowboarder should, in general, work to maximize his kinetic energy by applying as much pumping force as possible, while losing as little energy to friction and drag forces as he can.

Contents

1 Problem Introduction

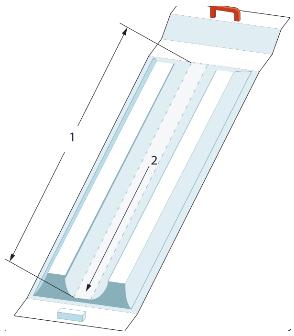
The goal of this report is to characterize the path and motion that a snowboarder should carry out to achieve the best score on the halfpipe. Previous work has been dedicated to researching how to create the optimal halfpipe course in order to allow the most exciting performances [1–3]. The problem I investigate here is, given the standard halfpipe course, how to best optimize the path of the snowboarder. The metric for a good path is a difficult one to choose, as scoring is infamously subjective in halfpipe competitions. Recent work has been done to use new technology to try to create an objective, standardized scoring system, such as evaluation based on tri-axial rate gyroscope data which involves a mounted inertial sensor [5].

The primary measurable features of a good performance are vertical air, maximum degree of rotation, maximum air time, average degree of rotation, and average air time [4–6]. A specific performance measure used previously to predict scores is the equation proposed by Harding et al.

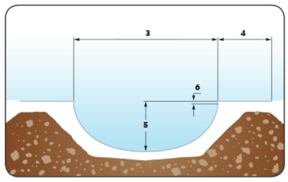
$$PredictedScore = 11.424(AAT) + 0.013(ADR) - 2.23 \tag{1.1}$$

Where AAT is the average air time and ADR is the average degree of rotation over all jumps [4]. The averages of air time and degree of rotation of course at least partially account for maximum air time and degree of rotation, making it a reasonable metric. The number of jumps that a snowboarder is allowed to perform in standard halfpipe competition such as the Olympics is 5-8 [1]. I assume it to be a constant of 5 jumps.

Standard dimensions of a snow halfpipe are given in Table 1 with the physical features



(a) Halfpipe Aerial View



(b) Halfpipe Front View

Figure 1: Graphic representation of the dimensions of a snow halfpipe [7]

	Feature	Dimension
1	Length of halfpipe	125m
2	Slope angle	16°
3	Width	18m
4	Width of side deck	7m
5	Height	5.4m
6	Height of pure vertical	.2m

Table 1: Dimensions of a Standard Snow Halfpipe [7]

numbered and diagrammed in Figure 1. The halfpipe is not technically a half cylinder as it would appear in 1b, however it is assumed to be.

In section 2 I develop the rigid body model of the snowboarder and introduce the inertia tensor. In sections 3 and section 4 I describe the motion in the air and in the halfpipe respectively. Then in section 5 I describe how to combine these models of motion in the air and on the halfpipe to describe a whole run down the halfpipe. In section 6 I use conservation of energy to analyze the problem of getting the maximum air time. Finally in section 7 I describe the mechanics of flipping and spinning, and how to maximize degree of rotation.

2 Rigid Body Model

In this section the model of the snowboarder is developed. The body and board of the snowboarder are assumed to constitute a rigid body, frozen in place as depicted in Figure 2. This is a strong assumption that disregards many of the more interesting features of snowboarding the halfpipe, and thus the rigid body assumption is relaxed in later analysis.

The center of mass of a multi-particle system, such as the rigid body of the snowboarder, can be described:

$$\overline{R}_{cm} = \frac{1}{M}\sum m_i \overline{r}_i = \frac{1}{M}\int \overline{r}dm \tag{2.1}$$

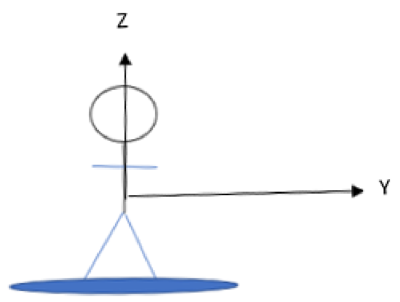


Figure 2: The rigid body representation of a snowboarder that will be used. Note the symmetry about the z-axis. The origin is the snowboarder's center of mass $\overline{R}_{cm}$.

Where the summation is replaced by an integral for a continuous distribution, which is more appropriate here. The center of mass of a human is typically around 10cm below the navel. For the snowboarder rigid body model, the center of mass is assumed to be lower than that, at waist level, due to the added mass of the snowboard.

The angular momentum of a system of particles with respect to the origin of the coordinate system can be written

$$\overline{L} = \sum_{\alpha} \overline{L}_{\alpha} = \sum_{\alpha} \left(\overline{r}_{\alpha} \times m_{\alpha} \dot{\overline{r}}_{\alpha} \right) \tag{2.2}$$

It can be shown that the angular momentum can be rewritten as

$$\overline{L} = \overline{L}_{cm} + \overline{L}_{wrt,cm} \tag{2.3}$$

Where $\overline{L}_{cm}$ is the angular momentum of the center of mass, and $\overline{L}_{wrt,cm}$ is the angular momentum of the body with respect to the center of mass. When the origin of the coordinate system is chosen to be the center of mass, the first term goes to 0.

The inertia tensor can be written as

$$\boldsymbol{I} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{22} & I_{33} \end{pmatrix} \tag{2.4}$$

Where the components are defined as

$$I_{ij} := \sum_{\alpha} m_{\alpha} \left(\|r^2\| \delta_{ij} - x_i x_j \right) \tag{2.5}$$

With the inertia tensor I, the components of the angular momentum can be written

$$L_i = \sum_{j} I_{ij} w_j \tag{2.6}$$

Referring to the rigid body model in Figure 2, it is immediately clear that there is symmetry about the z-axis. Thus, it is important to note, that one of the principal moments of inertia will be along this axis.

3 Motion in the Air

This section will explore the motion of the snowboarder in the air. Equations will be developed to state the required initial linear and angular velocity the snowboarder must take off the edge of the halfpipe with in order to preform m-spins before landing. This section will use the rigid body model, and thus only the motion of the center of mass is considered.

Consider the simple free body diagram of the snowboarder right before he takes off shown in Figure 3. In this diagram the x-axis is pointing down the slope of the halfpipe. The air time of a jump is only a function of the linear velocity in the z-direction at takeoff. The velocity in the x-direction is irrelevant, because the z axis is along the slope of the hill. The slope of the hill can be seen in Figure 4, where θ is 16°as given in Table 1. It also can be seen from Figure 4 that the components of gravity are given as $-g\cos(\theta)$ and $g\sin(\theta)$ along the z and y axes respectively.

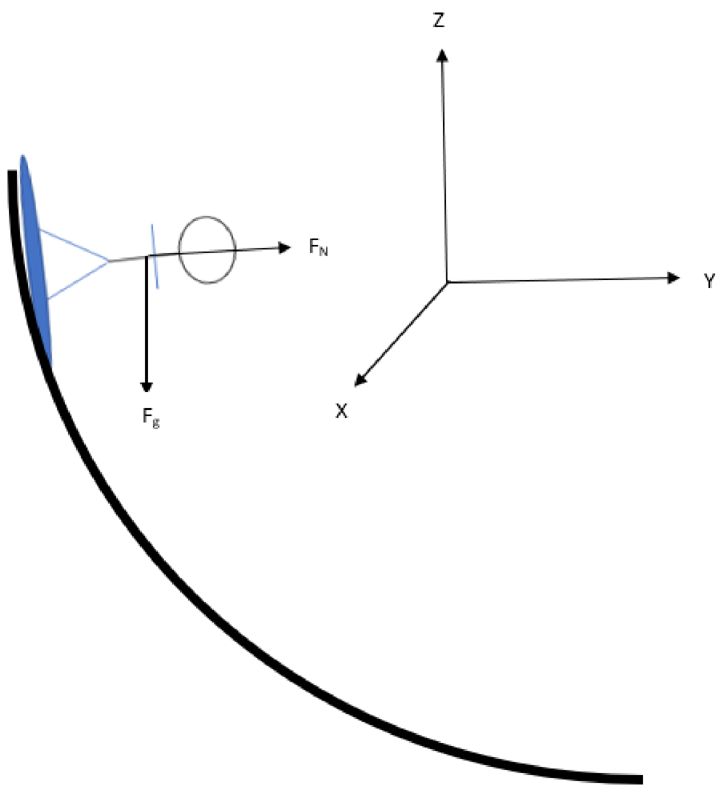


Figure 3: Free body diagram of the snowboarder before jump at the edge of the halfpipe. The origin is the snowboarder's center of mass $\overline{R}_{cm}$

Now we will find the position and velocity that the snowboarder will land with given the initial velocity and position. Looking at Figure 3, it seems reasonable to assume that there is no velocity in the y-direction at takeoff, so motion can be considered in only the

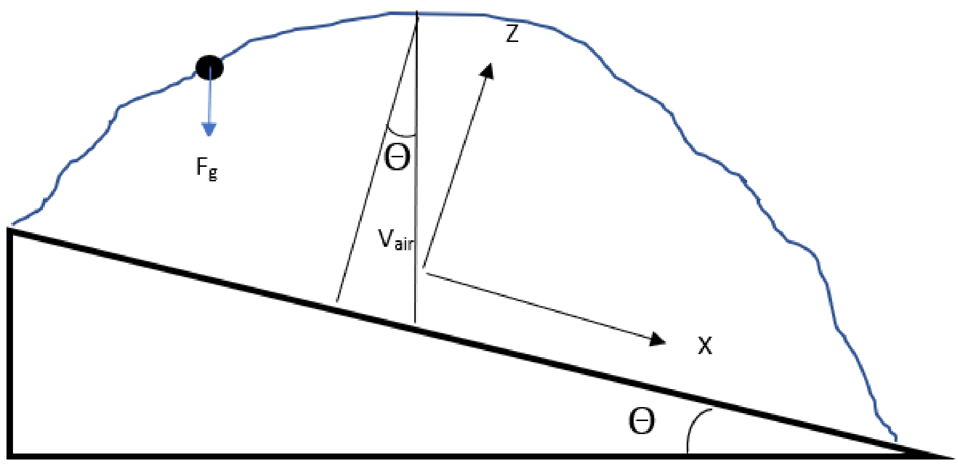


Figure 4: Slope of the hill, and path of snowboarder in the air along the x-z plane of the halfpipe.

x-z plane as depicted in Figure 4. The equation of motion of the snowboarder is given as

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = m\mathbf{g} - k\mathbf{v} \tag{3.1}$$

Where k is a drag coefficient and $\mathbf{g}$ is $(g\sin(\theta), 0, -g\cos(\theta))$. We can split 3.1 into component form as

$$\frac{\mathrm{d}\dot{x}}{\mathrm{d}t} = g\sin(\theta) - \frac{k}{m}\dot{x} \tag{3.2}$$

$$\frac{\mathrm{d}\dot{z}}{\mathrm{d}t} = -g\cos(\theta) - \frac{k}{m}\dot{z} \tag{3.3}$$

Integrating equations 3.2 and 3.3 using an integrating factor $e^{\int \frac{k}{m}dt}$ we get

$$\dot{x}(t) = \frac{mg\sin(\theta)}{k} + (v_{x0} - \frac{mg\sin(\theta)}{k})e^{\frac{-k}{m}t} \tag{3.4}$$

$$\dot{z}(t) = \frac{-mg\cos(\theta)}{k} + (v_{z0} + \frac{mg\cos(\theta)}{k})e^{\frac{-k}{m}t} \tag{3.5}$$

Integrating one last time to get the position as a function of time we get

$$x(t) = \frac{mg\sin(\theta)}{k}t - \frac{m}{k}\left(v_{x_o} - \frac{mg\sin(\theta)}{k}\right)e^{\frac{-k}{m}t} + C_1 \tag{3.6}$$

$$z(t) = -\frac{mg\cos(\theta)}{k}t - \frac{m}{k}\left(v_{z_o} + \frac{mg\cos(\theta)}{k}\right)e^{\frac{-k}{m}t} + C_2 \tag{3.7}$$

Where the constants C_1 and C_2 can be solved for using the initial position. For initial position $z(0) = 0$, and in the limit $t << m/k$ the result for z reduces to

$$z(t) = v_{z_o}t - \frac{1}{2}g\sin(\theta)t^2 \tag{3.8}$$

Which is what would be expected in the absence of drag, since this is just simple projectile motion with $g\cos(\theta)$ as the component of gravity in the z direction. The drag is assumed to be small enough that the total air time can be adequately approximated as

$$t_{air} = \frac{2v_{z_o}}{g\cos(\theta)} \tag{3.9}$$

Using this airtime, t_{air} , that has been solved for in terms of the components of takeoff velocity, the velocity upon landing can be calculated. Using t_{air} the angular momentum required for m-spins can be calculated as well. The snowboarder must land after completing $m \times 360°$ of rotation to complete m spins. If the rotation is assumed to be entirely about the y-axis of Figure 3, the angular velocity the snowboarder must take off with can be written in terms of air time as

$$w_y = \frac{2\pi m}{t_{air}} \tag{3.10}$$

However, this rotation of the snowboarder is not in general around solely the y-axis. Furthermore, the rigid body model of the snowboarder is a unrealistic one. Rotation in the air is explored more in 7.

4 Motion in the Halfpipe

The model used to describe the motion of the snowboarder on the ground is extremely simplified. The mechanical spring model used by Feng and Xin is adapted here to include friction and drag force, as well as a 3rd coordinate going down the hill [8]. This model is effective because it can account for the snowboarder doing work by pumping to increase his kinetic energy. Pumping in the halfpipe is analogous to pumping on a swing, where the rider in this case pushes against the normal force. First note that the motion of the snowboarder will be represented in cylindrical coordinates as shown in Figure 5, with the x coordinate coming out of the page, sloping down the hill.

The kinetic energy T can be calculated as

$$T = \frac{m}{2}\left(\dot{x}^2 + \dot{h}^2 + (R-h)^2\dot{\phi}^2\right) \tag{4.1}$$

And the potential energy is

$$U = -mg(R-h)\cos(\phi)\cos(\theta) - mgx\sin(\theta) \tag{4.2}$$

Where again the $\sin(\theta)$ and $\cos(\theta)$ terms come from taking the components of gravity along the respective axes as seen in Figure 4. The Lagrangian is defined as $L = T - U$. The Lagrange equations of motion are

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j \tag{4.3}$$

Where accounting for the forces of friction and drag in the Lagrange equations of motion involves adding a Q_j term for generalized forces, which can be done as long as the generalized forces satisfy D'Alembert's principle [10]. The friction force cannot be

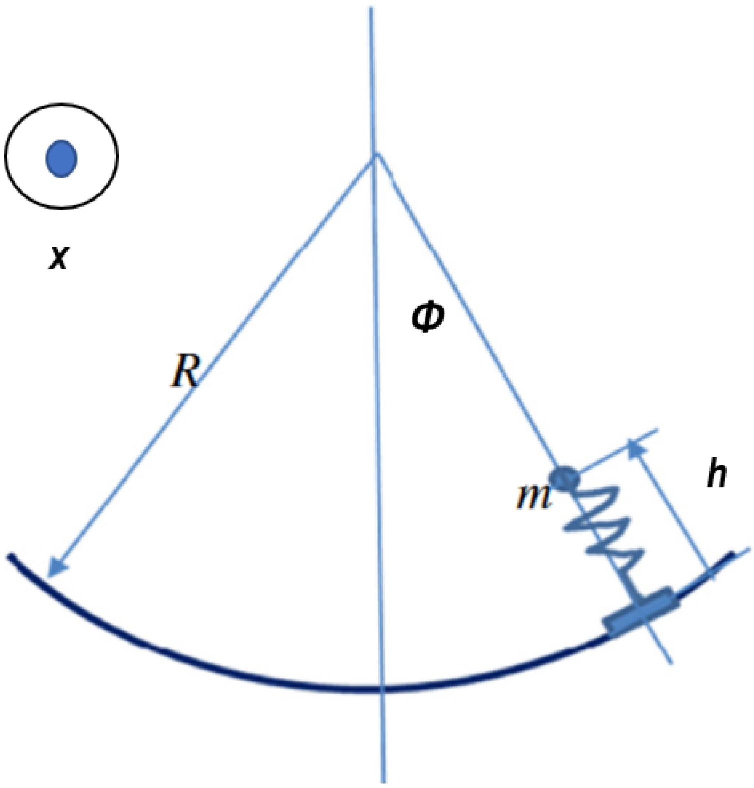


Figure 5: Mechanical model of a snowboarder, adapted from [8]. The generalized coordinates used will be ϕ, x, and h, where h is the distance to the snowboarder's center of mass.

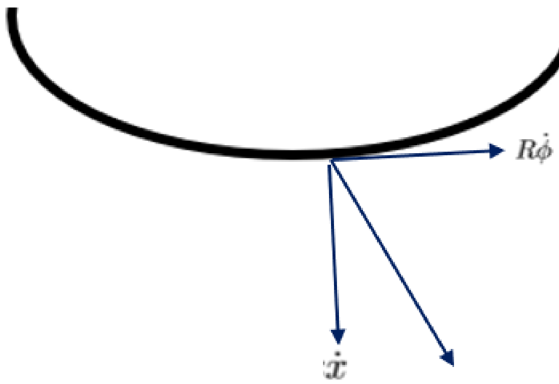


Figure 6: Shows the direction of motion and the components of velocity

accounted for in the Lagrangian itself because it is not a conservative force. Using the definition in 4.3, the Lagrange equation of motion for the ϕ coordinate becomes

$$m\left[(R-h)^2\ddot{\phi}-2(R-h)\dot{h}\dot{\phi}+g(R-h)\sin(\phi)\cos(\theta)\right]=-kR\dot{\phi}-\mu\frac{R\dot{\phi}}{\sqrt{(R\dot{\phi})^2+\dot{x}^2}}(mg\cos\theta+mR\dot{\phi}^2) \tag{4.4}$$

Using in this case $Q_\theta=-kR\dot{\phi}-\mu\frac{R\dot{\phi}}{\sqrt{(R\dot{\phi})^2+\dot{x}^2}}(mg\cos\theta+mR\dot{\phi}^2)$, where this formulation assumes a linear resistance from drag as the first term and a frictional force with constant μ as the second term. The $\frac{R\dot{\phi}}{\sqrt{(R\dot{\phi})^2+\dot{x}^2}}$ part of the friction term comes from taking the component of friction along the direction of as seen in Figure 6, and the other half of this term is the normal force due to gravity and centripetal acceleration. The Lagrange equation of motion for the h coordinate is given by

$$m\left[\ddot{h}+(R-h)\dot{\phi}^2-g\cos(\phi)\cos(\theta)\right]=-k_{spring}h+F_{rider} \tag{4.5}$$

Where the Q_j has been expressed using the mechanical model of a spring, with some driving force F_{rider} that the snowboarder is providing by pushing off the ground. Lastly the Lagrange equation of motion for the x coordinate is

$$m\ddot{x}+mg\sin(\theta)=-k\dot{x}-\mu\frac{\dot{x}}{\sqrt{(R\dot{\phi})^2+\dot{x}^2}}mg\cos\theta \tag{4.6}$$

Immediately you can recognize this equation for x is nearly the same as equation 3.2, with an additional term for the frictional force. These equations for ϕ, h, and x are not solved analytically and are rather used in Mathematica to simulate motion.

Now of course this model leaves many things out. The one that seems most blatantly relevant is the snowboarder's ability to convert velocity in the $\hat{\phi}$ direction to velocity in the $\hat{x}$ direction and vice versa, or more simply put, it ignores the ability to turn! The other important thing to keep in mind is that this Lagrangian is only applicable on the domain $-\pi/2<\phi<\pi/2$, where the snowboarder is on the ground as can be seen in Figure 5.

5 Motion Simulation

Now that we have a decent model of the motion that happens while the snowboarder is in the halfpipe, and the motion that happens when the snowboarder is in the air, what's left is to put them together. The starting idea is that we can begin by simulating the motion in the halfpipe to find the initial conditions of the motion in the air, simulate the motion in the air to find the initial conditions for subsequent motion in the halfpipe, and repeat for all 5 jumps.

The first set of initial conditions for motion in the halfpipe are taken to be when the snowboarder is starting from the top of the halfpipe cylinder with some initial velocity. The components of the initial velocity are taken as $R\dot{\phi}[0]=20m/s$, the component across the width of the halfpipe, and $v_x[0]=10m/s$, the component down the slope of the half-pipe. These values are based on rough estimation from YouTube videos. The snowboarder typically starts upright, so the initial conditions for h are assumed to be $h[0]=1.8m$ and

$h'[0] = 0$ corresponding to a 6 foot snowboarder standing upright. Equations 4.4, 4.5, and 4.6 were solved using NDSolve in Mathematica, with these initial conditions. Then to find the initial conditions of the air motion, note that the $\hat{z}$ component of velocity right after takeoff is equal to the $\hat{\phi}$ component of velocity right before takeoff, as can be seen in Figures 3 and 5. The $\hat{x}$ component of velocity remains the same, except now has no friction term, only drag force. The $\hat{y}$ component of velocity is assumed to be zero, though is typically some very small value. We then of course also have initial conditions for the position right before takeoff as well. Thus we can solve equations 3.6 and 3.7, with these initial conditions.

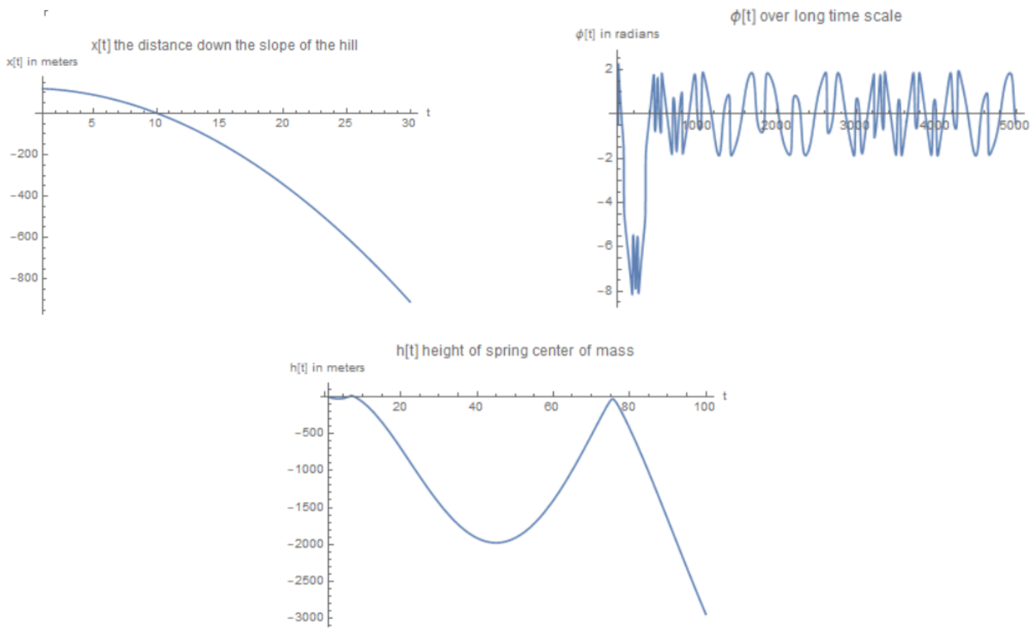


Figure 7: Plots to the solutions of equations 4.4, 4.5, and 4.6 with initial conditions described in Section 5.

Upon landing we can find the find the initial velocity in the $\hat{\phi}$ direction on the ground as the $\hat{z}$ component of velocity from being in the air, reversing the relation we used previously. The initial value of h is just assumed to be R/2 upon landing, as the human usually must bend his knees and compress his body to absorb impact. I attempted to iteratively use these continuity conditions and the equations of motion given in 3.6, 3.7, 4.4, 4.5, and 4.6 to solve for a path down the halfpipe. I expected to be able to construct a path from these piecewise functions similar to that seen in Figure 8.

Solving the equations of motion in the air was trivial. Solving the equations of motion on the ground proved difficult. Using the initial conditions described for the first segment of the path, and a zero pumping function F_{rider}, the solution to the coupled equations 4.4, 4.5, and 4.6 for $\phi[t]$, $h[t]$, and $x[t]$ can be seen in Figure 7. The graph of $x[t]$ is simple and reasonable, as it describes the position down the slope of the hill and is decreasing in an expected concave down manner. The graph of $\phi[t]$ is plotted over a large time scale to see the oscillatory behavior, which should be expected, and the initial steep slope downwards is on account of the initial velocity, which also seems reasonable. Now the plot of $h[t]$, which is supposed to indicate the height of the center of mass of the snowboarder, is where things go awry. Though initially the starting conditions are satisfied, the height quickly takes on an extreme negative value, which is evidently impossible, as that would indicate the snowboarder was buried in the snow.

To try to make the $h[t]$, the height of the center of mass, take on realistic values, I tried to vary F_{rider}, the pumping and human force that is pushing up off the ground. However, *no* values in the range of 0-10000N, which are all the values of force a human could realistically be able to apply, gave non-negative heights. After a thorough analysis of the derivation and an even more thorough analysis of the implementation of the model, I reach the conclusion that there is likely something flawed in the model itself, as in it is not a realistic way of looking at the snowboarder halfpipe system, or there was an error made in the derivation.

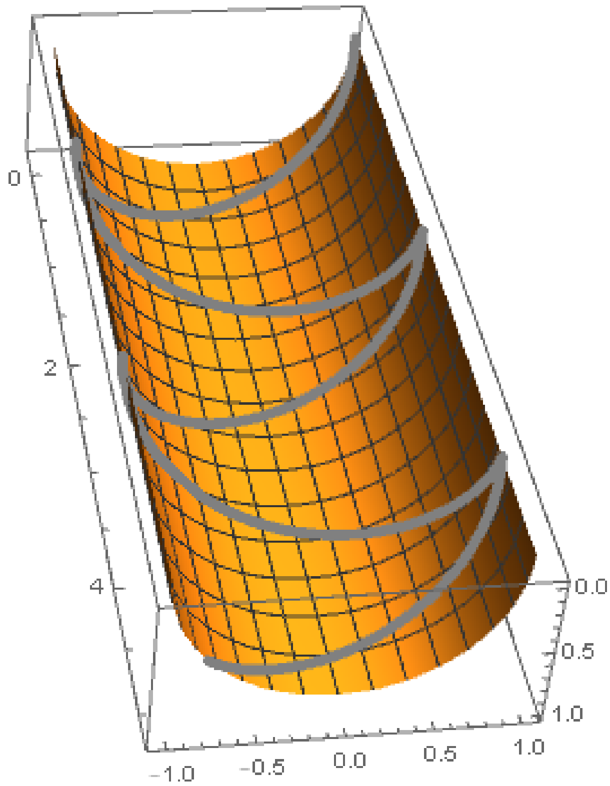


Figure 8: Attempted to solve the equations of motions derived with the continuity conditions mentioned, using DSolve in Mathematica. While in principal a realistic solution should have been found, none was. Instead this figure is a rough representation of what a path should have looked like.

6 Energy Analysis

We found in equation 3.9 the snowboarder wants to maximize his velocity along the z-axis in order to maximize his air time. Using the rigid body model, the kinetic energy in terms of cylindrical coordinates can be written

$$T = \frac{m}{2}\left(\dot{x}^2 + R^2\dot{\phi}^2\right) \tag{6.1}$$

We also argued that the $\hat{z}$ component of velocity right after takeoff is equal to the $\hat{\phi}$ component of velocity right before takeoff. Thus in order to have maximal air time, we are equivalently maximizing the $\hat{\phi}$ component of velocity. Now our model thus far has assumed that the rider is unable to turn, while incorporating turning into this model would clearly make it more realistic, there are some immediate difficulties with doing so. Imagine first if the snowboarder were given absolute control over turning ability, as in he could change direction as sharply as he wanted with no loss of energy. The optimal path for maximizing air time and degree of rotation in this case would be to quickly ride close to the bottom of the halfpipe, converting all the gravitational potential energy to kinetic energy. Then at this peak velocity at the bottom of the halfpipe, the snowboarder would want to turn all his velocity from the $\hat{x}$ direction into the $\hat{\phi}$ direction, as can be seen from the kinetic energy balance in 6.1. Then the rider would perform all jumps in a small a distance of each other, where he can have a maximal $\hat{z}$ component of velocity. Now realistically this is impossible, there is only *some* degree of control the snowboarder has over his turn radius, and typically there is some loss of kinetic energy involved in turning. This tradeoff varies from snowboarder to snowboarder and turn to turn.

Maximizing the kinetic energy before takeoff is clearly associated with longer air time as has been argued, and also enables more angular rotation as discussed in 7. Thus, how to maximize kinetic energy is explored here. We begin by writing energy conservation as

$$T_i + (U_i - U_f) + W_{rider} - E_{lost} = T_f \tag{6.2}$$

Let the subscript f denote the time right before any given jump, and the subscript i denote the time right after the previous jump. Then W_{rider} is the work the rider does by pumping between the jumps, and E_{lost} is the energy lost to friction and air drag between the jumps. Clearly in order to maximize T_f in 6.2 the rider will want to minimize E_{lost} and then maximize T_i, the change in potential energy $(U_i - U_f)$, and the work the rider does W_{rider}.

The initial kinetic energy in this case is 0 before the performance starts, and then just the final kinetic energy T_f of the *previous* jump in all other cases. Thus the term T_i is automatically maximized. Maximizing the change in potential energy term, $(U_i - U_f)$, is clearly indicating that the rider should ride as far down the hill as possible before performing his jumps. Now maximizing W_{rider} indicates the snowboarder should contribute as much pumping energy as possible during riding to maximize his kinetic energy and thus air time. Finally, E_{lost} to friction and drag can be found as

$$E_{lost} = \int_{r_i}^{r_f} (F_f + F_{drag})dr \tag{6.3}$$

The infinitesimal length dr can be found as

$$dr = \sqrt{1 + \frac{dy}{dx}^2 + \frac{dz}{dx}^2} dx \tag{6.4}$$

It can also be written in terms of either of the other two coordinates. While trying to minimize this integral expression of E_{lost}, there is the constraint

$$z^2 + y^2 = R \tag{6.5}$$

This constraint corresponds to the snowboarder having to stay on the halfpipe. Although since neither z nor y are known as functions of x, this formulation is of little use in finding an optimal path. But it is still worth keeping in mind when choosing a path to maximize airtime, the path should minimize energy lost to friction and drag. Another important note is that the coefficient of friction is in general not constant and changes depending on the snowboard's orientation to the ground. A coefficient of friction of 0.02 is commonly used for snowboard on snow, however when carving smoothly, this can be even lower, and when sliding this coefficient is much higher [9].

7 Rotation in the Air

Until now, rotation in the air has only been discussed briefly, and with a rigid body model. The key to understanding rotation in the air is a proper understanding of angular momentum. Angular momentum can be calculated as

$$L = I \cdot \omega \tag{7.1}$$

Angular momentum is always conserved during the flight of the snowboarder. Thus it is advantageous for the snowboarder to minimize his moment of inertia in order to maximize his angular velocity. The snowboarder can do this by tucking his body closely to the axis of rotation. For example in the case where the angular velocity is along the direction of one of the principal moments of inertia, like rotation about the y-axis in 9, the angular momentum reduces to

$$L = I_p \omega \tag{7.2}$$

Where I_p is the principal moment of inertia about that axis of rotation. For a fixed angular momentum, and changing principal moment of inertia, we can write

$$w_2 = \frac{I_{p1}}{I_{p2}} w_1 \tag{7.3}$$

So clearly in this case for the snowboarder to maximize rotation, it is best for him to contort his body to have the smallest moment of inertia about that axis, a result that holds in general as well.

The snowboarder's rotation about the y and z axes, includes tricks typically called "spins" or "corks" where corks just refer to spins during which the snowboarder inverts or

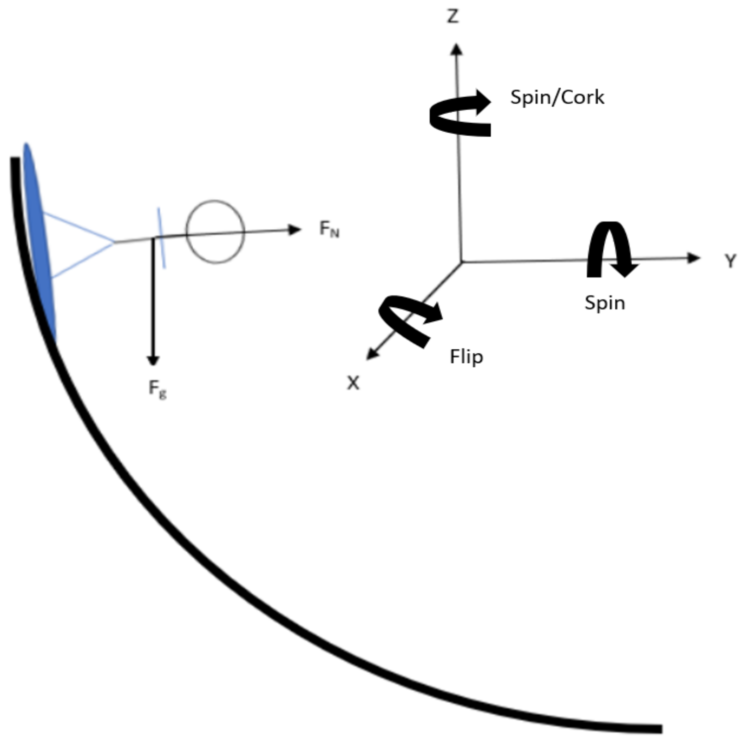


Figure 9: Tricks can be performed by rotation about various axes, with rotation generated by either angular velocity from takeoff or contortion of the body midair

turns his body sideways during the spin [9]. A full rotation about the x axis is called a flip. Generating rotation is arguably the most technical part of snowboarding the halfpipe. During takeoff, the snowboarder can easily convert velocity in the $\hat{\phi}$ into angular rotation about the x axes, and can generate additional angular velocity by leaning backwards, shifting his center of mass and producing a moment this way. Generating rotation about the y and z axes is largely from winding up the upper body before takeoff, and then sliding/whipping either the backedge or frontedge of the board around at takeoff. The energy of this rotation can come from the rider leaping and twisting to generate torque, or from converting linear momentum into angular momentum. The energy of rotation is given by

$$E_{rot} = \frac{1}{2} I \cdot \omega^2 \tag{7.4}$$

In addition to rotation from angular momentum at takeoff, during flight the human body can in fact end up spinning around axes with no initial angular momentum, or flip and spin without any initial rotation at all [11]. It has even been shown that in fact even a full flip can be done under the right conditions with no angular momentum at all, simply by reorienting the body in the proper way midair [11]. This emphasizes how the rigid body model is not enough to capture the complex motion of a snowboarder. In general, a model of several actuated joints, with many degrees of freedom is used to represent the human body's complex aerial motion, however this type of analysis will not be replicated here.

8 Conclusion

This report analyzed a few features of snowboarding the halfpipe, with the most thorough analysis on how to maximize air time. The model of pumping in the halfpipe was developed using the Lagrangian, incorporating non conservative forces. The model of translational motion in the air was akin to projectile motion with a drag force, and the model of the motion on the snow in the halfpipe was developed for a spring like body. Finally, a brief overview of some of the mechanics behind performing tricks in the air was discussed.

There were several weaknesses of this analysis. The first was that the rigid body model was thoroughly overused. Another big weakness was that the equations of motion developed were not able to be solved with a realistic solution. Other shortcomings included that no effective description of turning was incorporated, and the aerial trick motion was only explored to a limited extent.

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